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 $-82 \sin^3\delta \cos \delta + 32 \sin^5\delta \cos \delta + 96 \sin(\theta - \delta) \sin\theta \sin\delta (\delta - \sin\delta \cos\delta)^2 ] \sin^2\theta \sin^2\delta d\theta d\delta$ 

$$=\frac{5a^{2}}{162\pi^{4}}\int_{0}^{\pi}(360\theta^{2}-144\theta^{3}\sin\theta\cos\theta-360\theta^{2}\sin^{4}\theta+252\theta^{2}\sin^{2}\theta+336\theta\sin^{6}\theta\cos\theta$$

 $-132\theta\sin^3\cos\theta - 615\theta\sin\theta\cos\theta + 72\sin^3\theta - 250\sin^6\theta - 301\sin^4\theta + 255\sin^2\theta\sin^2\theta d\theta$ 

$$= \frac{5a^2}{12\pi} \Big( 5 - \frac{1001}{144\pi^2} \Big).$$

## MISCELLANEOUS.

116. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics. Curry University, Pittsburg, Pa.

Prove that
$$\begin{vmatrix}
a & b & c & a_1 \\
b & d & e & a_2 \\
c & e & f & a_3 \\
a_1 & a_2 & a_3 & 0
\end{vmatrix}
\begin{vmatrix}
a & b & c & a_1 \\
b & d & e & a_2 \\
c & e & f & a_3 \\
\beta_1 & \beta_2 & \beta_3 & 0
\end{vmatrix}
\begin{vmatrix}
a & b & c & a_1 \\
b & d & e & a_2 \\
c & e & f & a_3 \\
\gamma_1 & \gamma_2 & \gamma_3 & 0
\end{vmatrix}$$

$$- |a^2 \beta^2 \gamma^2|^2 |a^2|^2 |a^2|^2$$

[From Muir's Determinants.]

Solution by G. B. M. ZERR, A.M.. Ph.D., Professor of Chemistry and Physics, The Temple College. Philadelphia, Pa.

Let x, y, z be the minors with respect to  $\alpha_1, \alpha_2, \alpha_3$  in the first row, u, v, w the minors with respect to  $\beta_1, \beta_2, \beta_3$  in the second row, and r, s, t the minors with respect to  $\gamma_1, \gamma_2, \gamma_3$  in the third row of the determinant  $\triangle$  on the right hand side of the equality. Then

$$= - \begin{vmatrix} a_{1}x - a_{2}y + a_{3}z, & \beta_{1}x - \beta_{2}y + \beta_{3}z, & \gamma_{1}x - \gamma_{2}y + \gamma_{3}z \\ a_{1}u - a_{2}v + a_{3}w, & \beta_{1}u - \beta_{2}v + \beta_{3}w, & \gamma_{1}u - \gamma_{2}v + \gamma_{3}w \\ a_{1}r - a_{2}s + a_{3}t, & \beta_{1}r - \beta_{2}s + \beta_{3}t, & \gamma_{1}r - \gamma_{2}s + \gamma_{3}t \end{vmatrix}$$

$$= \begin{vmatrix} a_{1} & a_{2} & a_{3} \\ \beta_{2} & \beta_{2} & \beta_{3} \\ \gamma_{1} & \gamma_{2} & \gamma_{3} \end{vmatrix} \begin{vmatrix} x & y & z \\ u & v & w \\ r & r & t \end{vmatrix} = (a_{1}\beta_{2}\gamma_{3}) \begin{vmatrix} x & y & z \\ u & v & w \\ r & s & t \end{vmatrix}$$

$$=(a_{1}\beta_{2}\gamma_{3}) \left| \begin{array}{c|ccc} b & c & a_{1} \\ d & e & a_{2} \\ e & f & a_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & c & a_{1} \\ b & e & a_{2} \\ c & f & a_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & b & a_{1} \\ b & d & a_{2} \\ c & e & a_{3} \end{array} \right|$$

$$=(a_{1}\beta_{2}\gamma_{3}) \left| \begin{array}{c|ccc} b & c & \beta_{1} \\ d & e & \beta_{2} \\ e & f & \beta_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & c & \beta_{1} \\ b & e & \beta_{2} \\ c & f & \beta_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & b & \beta_{1} \\ b & d & \beta_{2} \\ c & e & \beta_{3} \end{array} \right|$$

$$\left| \begin{array}{c|ccc} b & c & \gamma_{1} \\ d & e & \gamma_{2} \\ e & f & \gamma_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & c & \gamma_{1} \\ b & e & \gamma_{2} \\ c & f & \gamma_{3} \end{array} \right| \left| \begin{array}{c|ccc} a & b & \gamma_{1} \\ b & d & \gamma_{2} \\ c & e & \gamma_{3} \end{array} \right|$$

Let A, B, C, etc., be the minors with respect to a, b, c, etc. Then

$$\left|egin{array}{cccc} a&b&c\b&d&e\c&e&f \end{array}
ight|^2=\left|egin{array}{cccc} A&-B&C\c&D&-E\c&C&-E&F \end{array}
ight|$$

$$\therefore \triangle = (a_1 \beta_2 \gamma_3) \quad \begin{vmatrix} a_1 A - a_2 B + a_3 C, & a_1 B - a_2 D + a_3 E, & a_1 C - a_2 E + a_3 F \\ \beta_1 A - \beta_2 B + \beta_3 C, & \beta_1 B - \beta_2 D + \beta_3 E, & \beta_1 C - \beta_2 E + \beta_3 F \\ \gamma_1 A - \gamma_2 B + \gamma_3 C, & \gamma_1 B - \gamma_2 D + \gamma_3 E, & \gamma_1 C - \gamma_2 E + \gamma_3 F \end{vmatrix}$$

$$= - (a_1 \beta_2 \gamma_3)^2 \begin{vmatrix} A & -B & C \\ -B & D & -E \\ C & -E & F \end{vmatrix} = - (a_1 \beta_2 \gamma_3)^2 \begin{vmatrix} a & b & c \\ b & d & e \\ c & e & f \end{vmatrix}.$$

117. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If  $x\cos a + y\cos a = a\cos \theta + b\cos \varphi$ , and  $x\sin a + b\sin \varphi = y\sin a + a\sin \theta = \kappa$ , find the maximum value of  $\kappa$ , and the values of x and y.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College. Philadelphia, Pa.

$$(x+y)\cos a = a\cos \theta + b\cos \varphi$$
....(1).  
 $(x+y)\sin a + a\sin \theta + b\sin \varphi = 2\kappa$ ....(2).

(1) in (2) gives

$$\frac{(a\cos\theta+b\cos\varphi)\sin\alpha}{\cos\alpha}+a\sin\theta+b\sin\varphi=2\kappa$$
, or

$$a\sin(\theta+a)+b\sin(\varphi+a)-2\kappa\cos a=0=u.$$

- $\therefore du/d\theta = a\cos(\theta + a) = 0, du/d\varphi = b\cos(\varphi + a) = 0.$
- $\therefore \theta = \varphi = \frac{1}{2}\pi a$  for a maximum.
- $\therefore \kappa = \frac{1}{2}(a+b)\sec a$  is the maximum value.

$$x\cos a + \frac{(\kappa - a\sin\theta)\cos a}{\sin a} = a\cos\theta + b\cos\varphi.$$